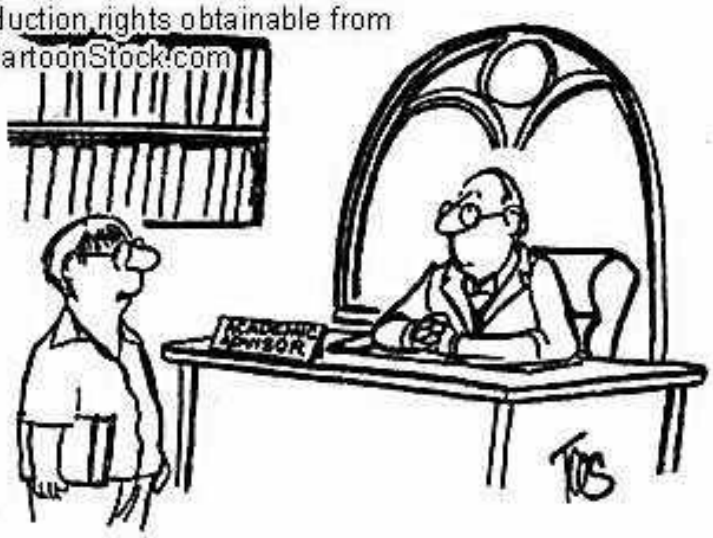
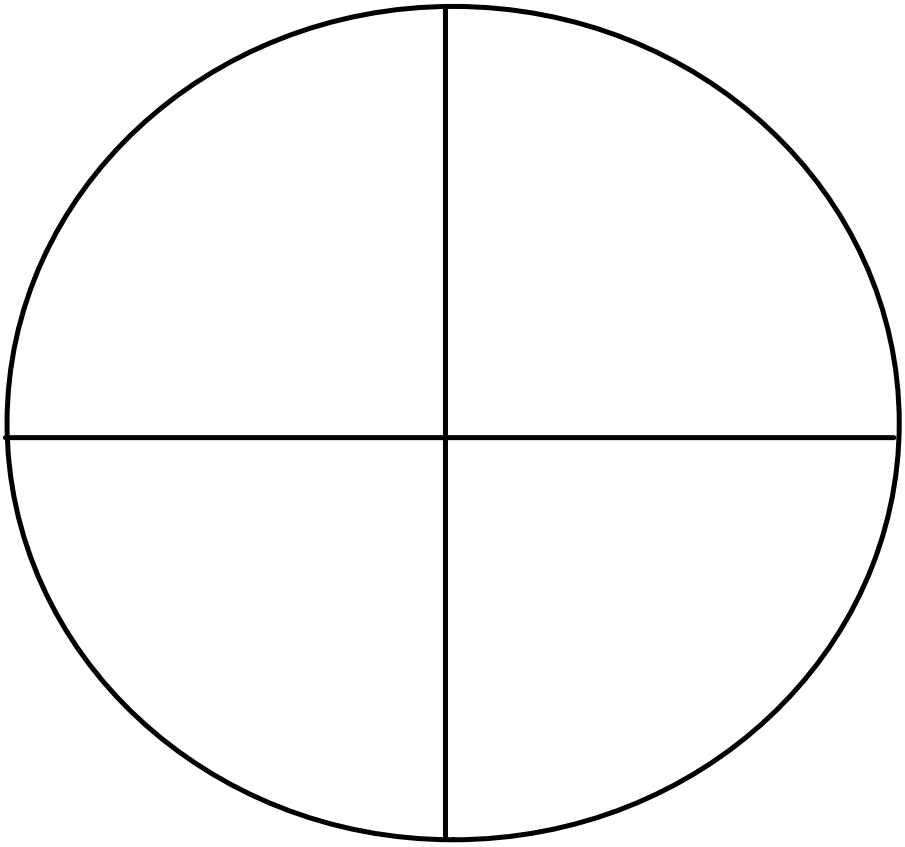
6.3 Trig. Functions of Any Angle



"I've decided to forego trigonometry,
and make myself eligible for the NBA draft."



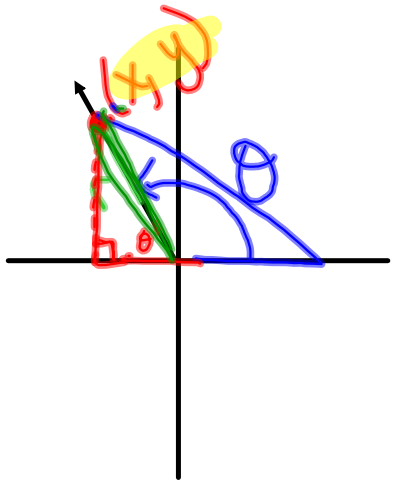
Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2+y^2}$

$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$

$$\cos\theta = \frac{x}{r} \qquad \sec\theta = \frac{r}{x}$$

$$\tan\theta = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$

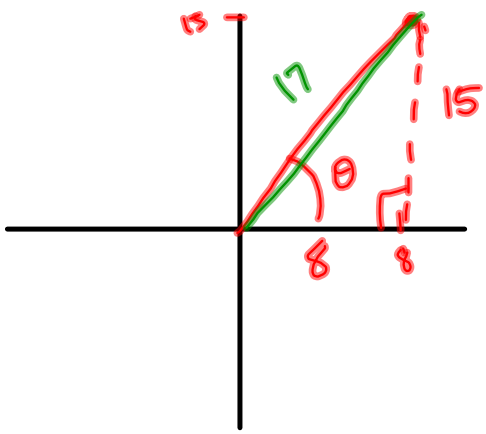


If the point (8, 15) lies on the terminal side of an angle in standard position, then determine the exact values of the six trigonometric values.

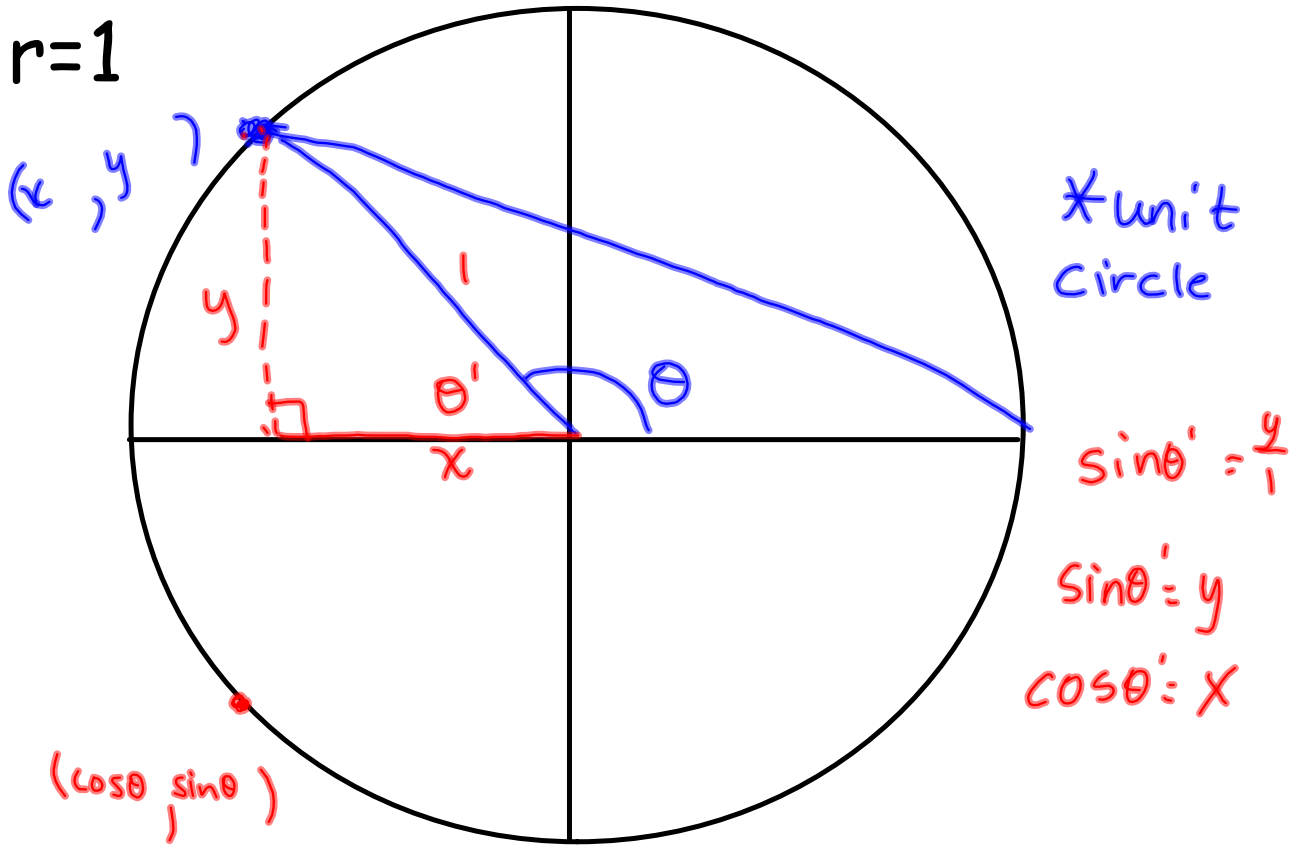
$\sin\theta = \frac{15}{17}$

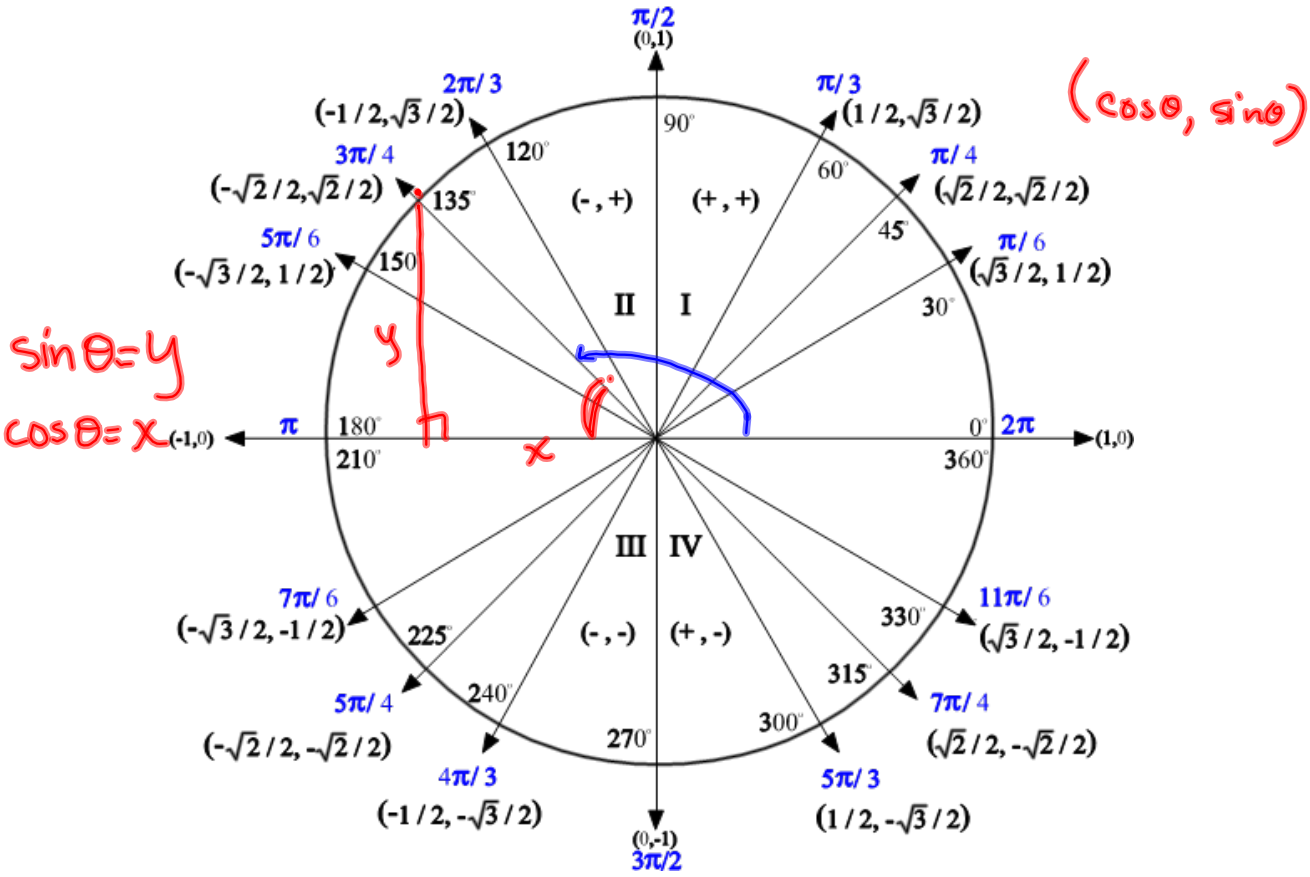
$\cos\theta = \frac{8}{17}$

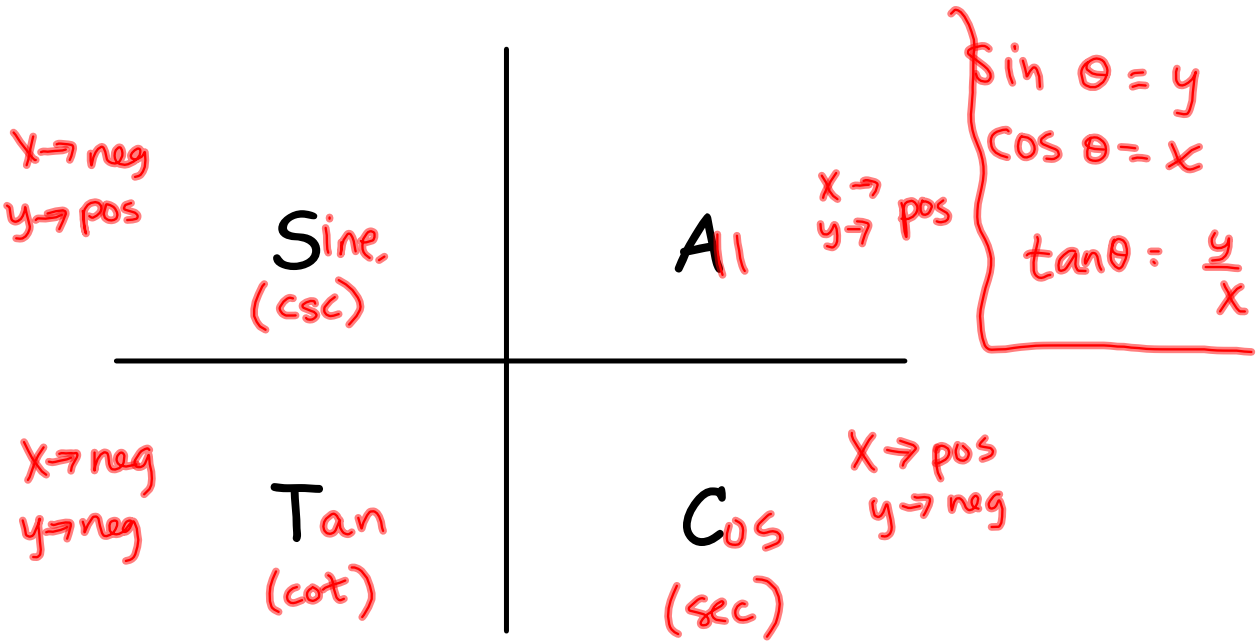
$\tan\theta = \frac{15}{8}$



$15^2 + 8^2 = r^2$

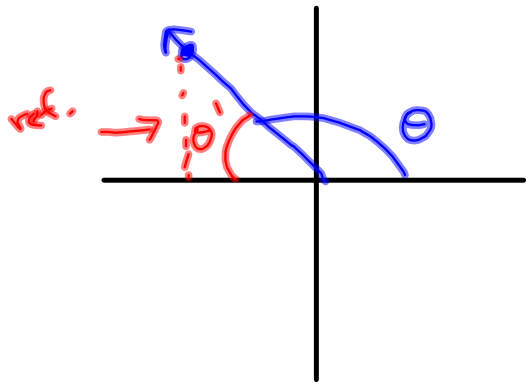

r=1
(x , y)
y
1
θ'
θ
x
*unit circle
sinθ' = y/1
sinθ' = y
cosθ' = x
(cosθ , sinθ)


π/2
(0,1)
2π/3
(-1/2,√3/2)
π/3
(1/2,√3/2)
(cosθ, sinθ)
3π/4
(-√2/2,√2/2)
π/4
(√2/2,√2/2)
5π/6
(-√3/2, 1/2)
π/6
(√3/2, 1/2)
90°
120°
60°
135°
45°
150°
30°
(-, +)
(+, +)
II
I
sinθ=y
y
cosθ=x
(-1,0)
π
180°
210°
x
0°
2π
(1,0)
360°
III
IV
7π/6
(-√3/2, -1/2)
11π/6
(√3/2, -1/2)
225°
330°
(-, -)
(+, -)
315°
5π/4
(-√2/2, -√2/2)
240°
7π/4
(√2/2, -√2/2)
300°
270°
4π/3
(-1/2, -√3/2)
5π/3
(1/2, -√3/2)
(0,-1)
3π/2


x → neg
y → pos
Sine
(csc)
All
x →
y → pos
sin θ = y
cos θ = x
tanθ = y/x
x → neg
y → neg
Tan
(cot)
Cos
(sec)
x → pos
y → neg

Reference Angles

Let θ be an angle in standard position. Its **reference angle** θ is the acute angle formed by the terminal side of θ and the x-axis.

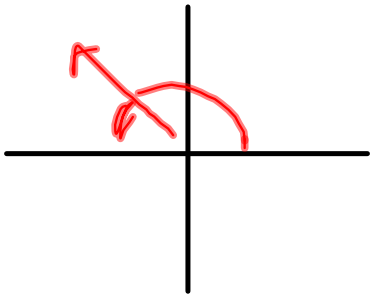


Always Positive!

Find the reference angle for the following angles.

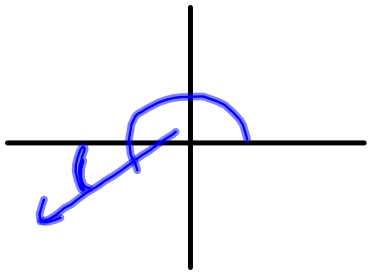
a) $\theta = 120°$

= 60°



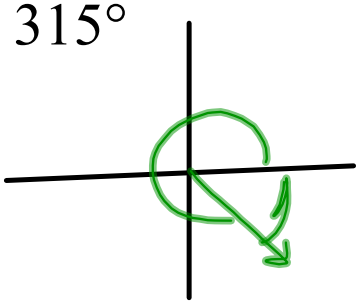
b) $\theta = 210°$

30°



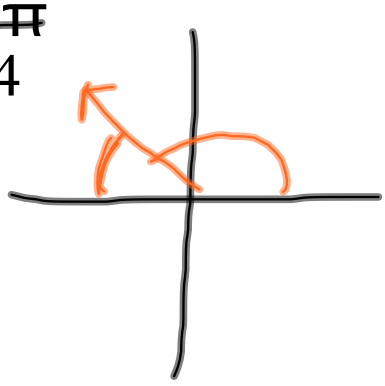
c) $\theta = 315°$

45°

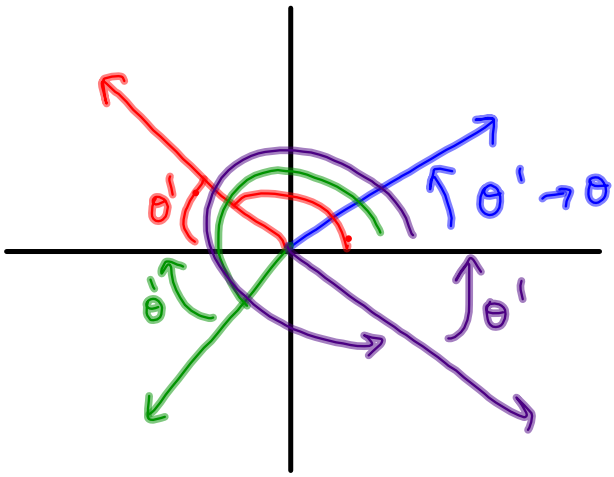


d) $\theta = \frac{3\pi}{4}$

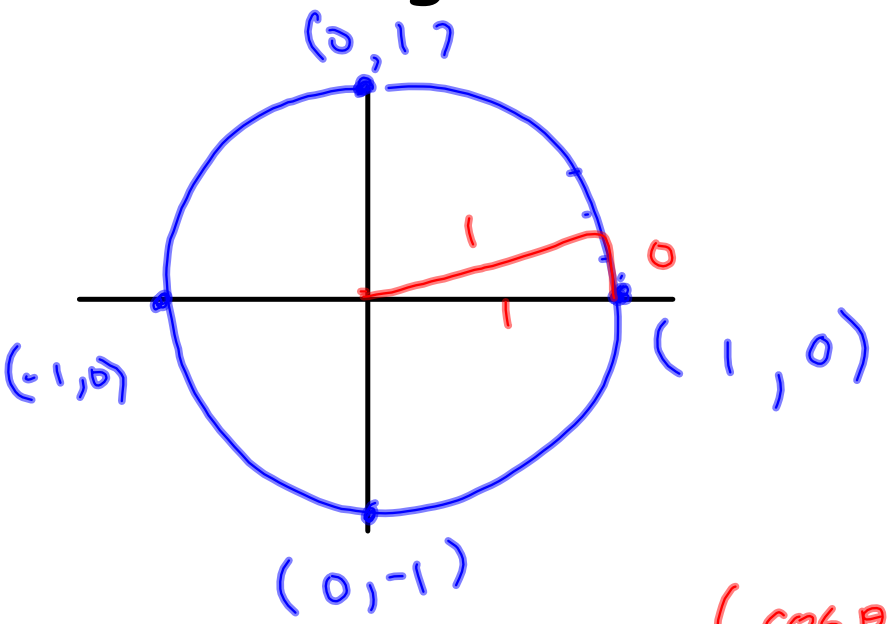
$\frac{\pi}{4}$



Reference angles for θ in Quadrants II, III, and IV



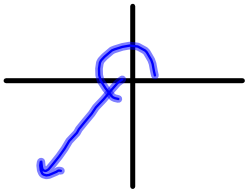
Quadrantal angles:



$(\cos\theta, \sin\theta)$

Ex 4 Evaluate the following trigonometric functions.

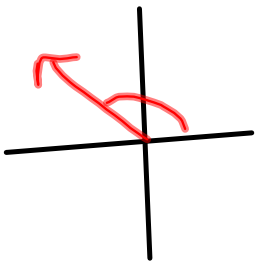
a) sin 225° = sin 45° = $-\frac{\sqrt{2}}{2}$



ref $\theta \to 45°$

neg

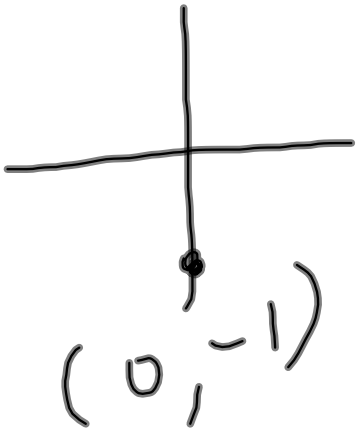
b) cos 120° = cos 60° = $-\frac{1}{2}$



ref $\theta \to 60°$

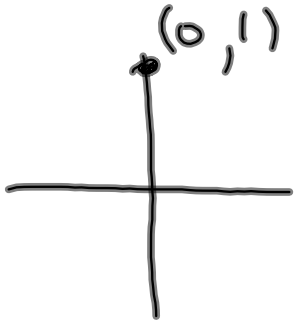
neg.

c) $\sin \frac{3\pi}{2} = -1$



$(0, -1)$

$\tan \frac{\pi}{2} = \varnothing$



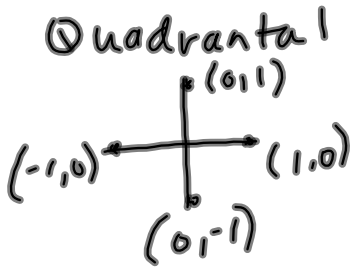
① Quadrant?

S | A
T | C pos/neg

② reference θ

$30^\circ, 60^\circ, 45^\circ$

$\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}$



③ Memorize trig values

- chart
- triangles

Domain of sine and cosine:

angle

all $\mathbb{R}$'s

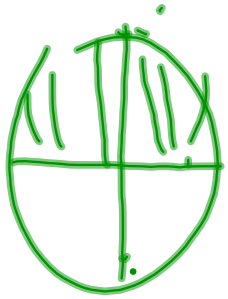
$\sin\theta = \frac{opp}{hyp}$

$\cos\theta = \frac{adj}{hyp}$

Range of sine and cosine:

ratio

$-1 \leq R \leq 1$



Domain of tangent

all $\mathbb{R}$'s $\quad \theta \neq \frac{\pi}{2} + \pi n$ $\quad \frac{opp}{adj}$

Range

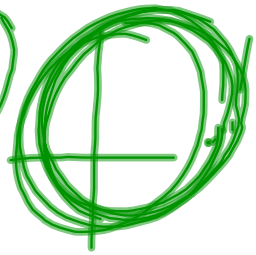
all $\mathbb{R}$'s

Periodic:

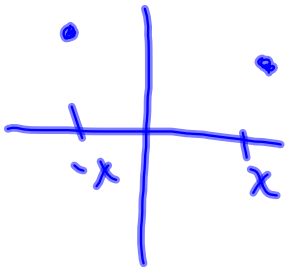
repeat every 2π

Evaluate $\sin \frac{13\pi}{6}$ using its period as an aid.

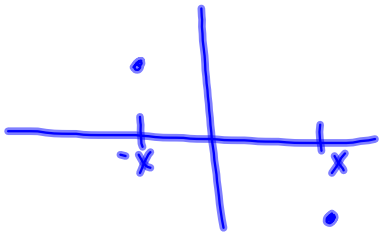
$\sin \frac{13\pi}{6} - \frac{12\pi}{6} = \sin \frac{\pi}{6}$



If f(-x) = f(x) then the function is even.



If f(-x) = -f(x) then the function is odd.



$\cos 60° = \frac{1}{2}$ $\cos -60° = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$ $\sin -30° = -\frac{1}{2}$

Cosine and secant functions are even.

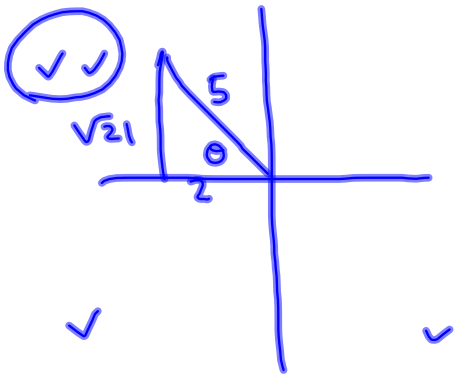
Sine, cosecant, tangent, and cotangent functions are odd.

$\cos t = -3/4$

$\cos(-t) = -\frac{3}{4}$

$\sec(-t) = -\frac{4}{3}$

Given $\cos\theta = -2/5$ and $\tan\theta < 0$, find the values of the six trigonometric functions of θ.



$\sin\theta = \frac{\sqrt{21}}{5}$

$\tan\theta = -\frac{\sqrt{21}}{2}$